Evaluation of Full Reynolds Stress Turbulence Models in FUN3D

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Objective

Evaluate two Reynolds-stress turbulence models (RSMs) available in the FUN3D unstructured CFD code: the SSG/LRR RSM and the Wilcox RSM. This work supports NASA's Revolutionary Computational Aerosciences (RCA) Technical Challenge:

Identify and down-select critical turbulence, transition, and numerical method technologies for 40% reduction in predictive error against ***standard test cases*** *for turbulent separated flows, evolution of free shear flows and shock-boundary layer interactions on state-of-the-art high performance computing hardware.*

Overview

- The FUN3D code
- The turbulence models
- Test cases – simple yet contain relevant flow physics
  - Transonic diffuser
  - Supersonic axisymmetric compression corner
  - Compressible planar shear layer
  - Subsonic axisymmetric jet
- Summary and conclusions

The FUN3D Code

- General purpose flow solver and design tool
- Developed by NASA Langley
- Wide variety of numerical schemes, gas models, turbulence models and boundary conditions
- Unstructured grids
- 2nd-order finite volume, node-centered
- Roe scheme (default)
  - Other methods available
- SA, SST-V, SSG/LRR RSM and Wilcox RSM used
- fun3d.larc.nasa.gov

The Wind-US Code

- General purpose flow solver
- Developed and supported by NASA Glenn, the Arnold Engineering Development Center (AEDC), The Boeing Co.
- Structured and unstructured grids
- 2nd-order accurate finite volume, node-centered, Roe (structured) and HLLE(unstructured) – default
- SA, SST-V, EASM models used
- www.grc.nasa.gov/winddocs

Turbulence Models

turbmodels.larc.nasa.gov

- Spalart-Allmaras (SA) one-equation model
  - Standard incompressible version
  - No trip term
  - $\tilde{\nu} / \nu = 5$ freestream boundary condition

- Menter's shear-stress transport (SST-V) two-equation model
  - Vorticity-based production term

- Two-equation explicit algebraic Reynolds stress model (EASM) (shear layer case)
  - Derived from reduced form of Reynolds stress transport equations
  - Similar to the Boussinesq approximation but includes terms that are nonlinear in the strain and rotation rate tensors

- Seven-Equation Omega-Based Full Reynolds Stress Turbulence Models
  - Wilcox Stress-Omega Full Reynolds Stress Model (Wilcox RSM)
  - SSG/LRR-Omega Full Reynolds Stress Model (SSG/LRR RSM)

Turbulence Models, cont'd

Seven-equation omega-based full Reynolds Stress models

SSG/LRR-Omega Full Reynolds Stress Model:

$$\tau_{ij} \stackrel{\text{def}}{=} \overline{-u_i''u_j''}$$

6 Reynold's Stress Equations and 1 Length Scale Equation:

$$\frac{\partial(\bar{\rho}\tau_{ij})}{\partial t} + \frac{\partial(\bar{\rho}\tau_{ij}\tilde{u}_k)}{\partial x_k} = -\bar{\rho}P_{ij} - \bar{\rho}\Pi_{ij} + \bar{\rho}\varepsilon_{ij} - \bar{\rho}D_{ij} - \bar{\rho}\mathcal{M}_{ij}$$

$$\frac{\partial(\bar{\rho}\omega)}{\partial t} + \frac{\partial(\bar{\rho}\omega\tilde{u}_k)}{\partial x_k} = \alpha_\omega \frac{\omega}{\tilde{k}}\frac{\bar{\rho}P_{kk}}{2} - \beta_\omega\bar{\rho}\omega^2 + \frac{\partial}{\partial x_k}\left[\left(\bar{\mu} + \sigma_\omega\frac{\bar{\rho}\tilde{k}}{\omega}\right)\frac{\partial\omega}{\partial x_k}\right] + \sigma_d\frac{\bar{\rho}}{\omega}\max\left(\frac{\partial\tilde{k}}{\partial x_k}\frac{\partial\omega}{\partial x_k}, 0\right)$$

- Blended Speziale-Sarkar-Gatski/Launder-Reece-Rodi pressure-strain model

$$\Pi_{ij} = -\left(C_1\varepsilon + \frac{1}{2}C_1^*P_{kk}\right)\tilde{a}_{ij} + C_2\varepsilon\left(\tilde{a}_{ik}\tilde{a}_{kj} - \frac{1}{3}\tilde{a}_{kl}\tilde{a}_{kl}\delta_{ij}\right) + \left(C_3 - C_3^*\sqrt{\tilde{a}_{kl}\tilde{a}_{kl}}\right)\tilde{k}\tilde{S}_{ij}^*$$
$$+ C_4\tilde{k}\left(\tilde{a}_{ik}\tilde{S}_{jk} + \tilde{a}_{jk}\tilde{S}_{ik} - \frac{2}{3}\tilde{a}_{kl}\tilde{S}_{kl}\delta_{ij}\right) + C_5\tilde{k}\left(\tilde{a}_{ik}\widetilde{W}_{jk} + \tilde{a}_{jk}\widetilde{W}_{ik}\right)$$

Wilcox Stress Omega Full Reynolds Stress Model:

- Uses a Launder-Rodi-Reece pressure-strain model

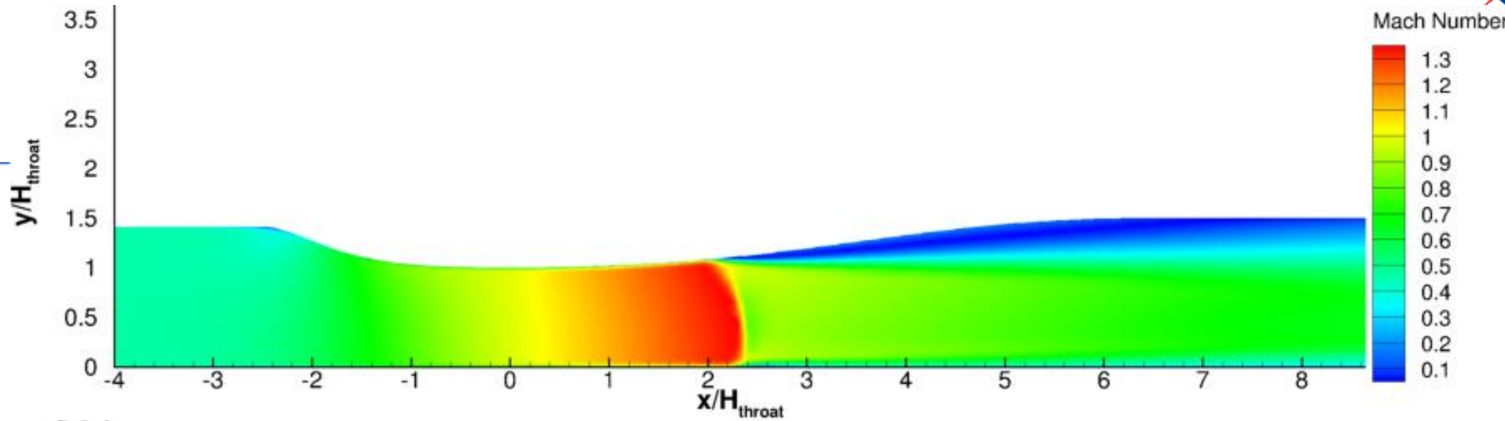
Overview

- The FUN3D and Wind-US codes
- The SSG/LRR and Wilcox Full Reynolds stress models
- Test cases – simple yet contain relevant flow physics
  - Transonic diffuser
  - Supersonic axisymmetric compression corner
  - Compressible planar shear layer
  - Subsonic axisymmetric jet
- Summary and conclusions

Test Cases

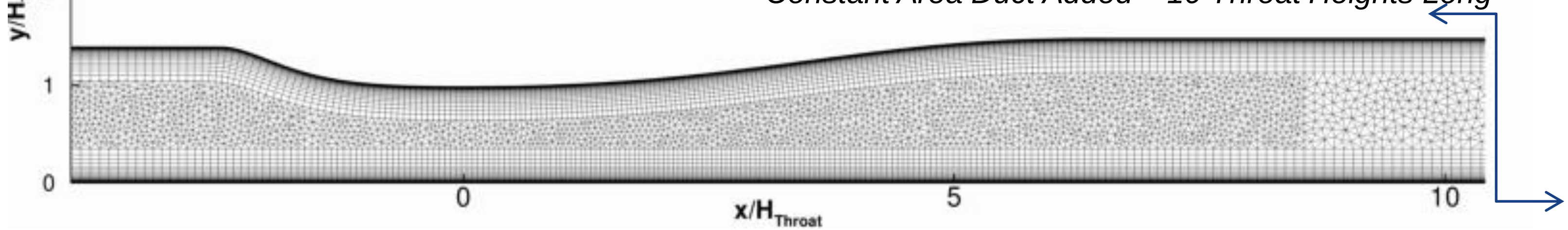
Transonic Diffuser – Strong Shock Case

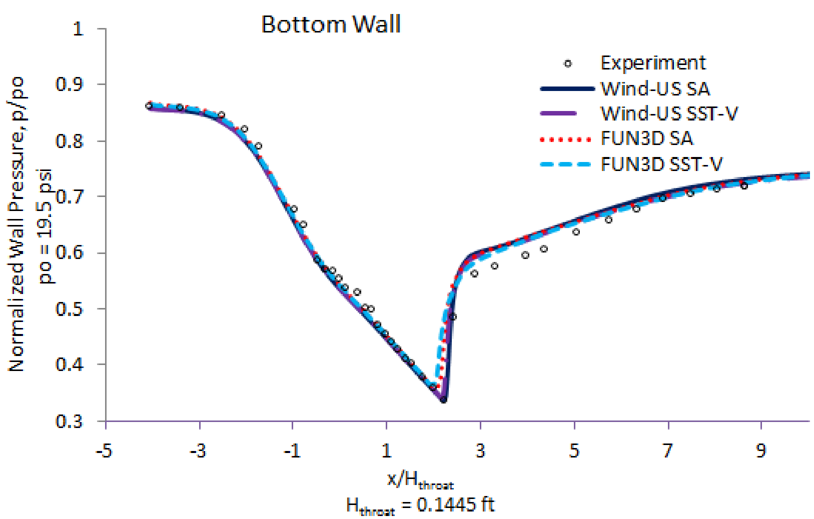
M=0.9
p=19.58 psi
T = 540 R

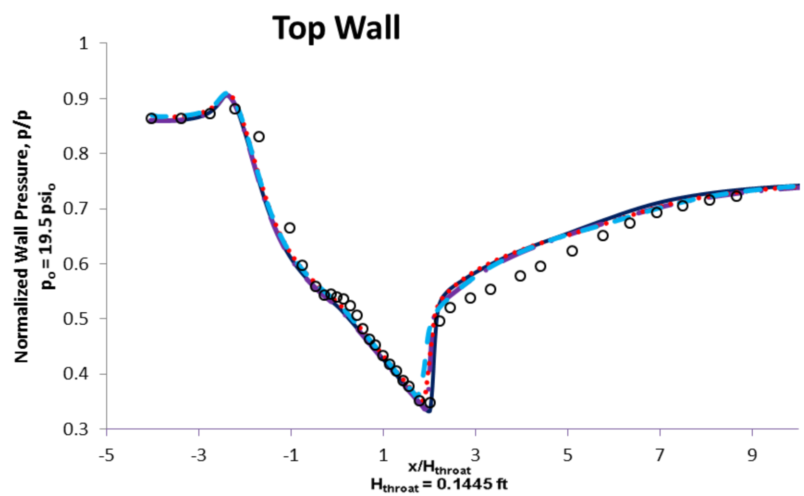


54,854 Grid Points

Constant Area Duct Added – 10 Throat Heights Long

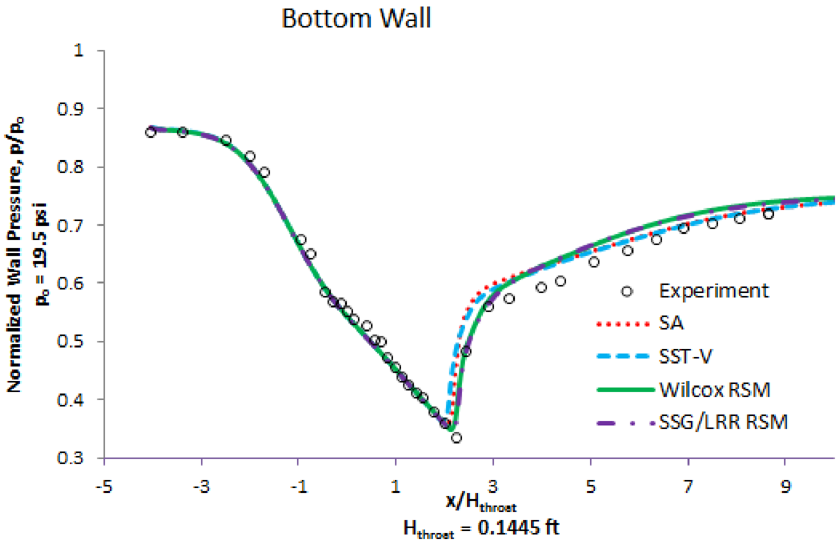


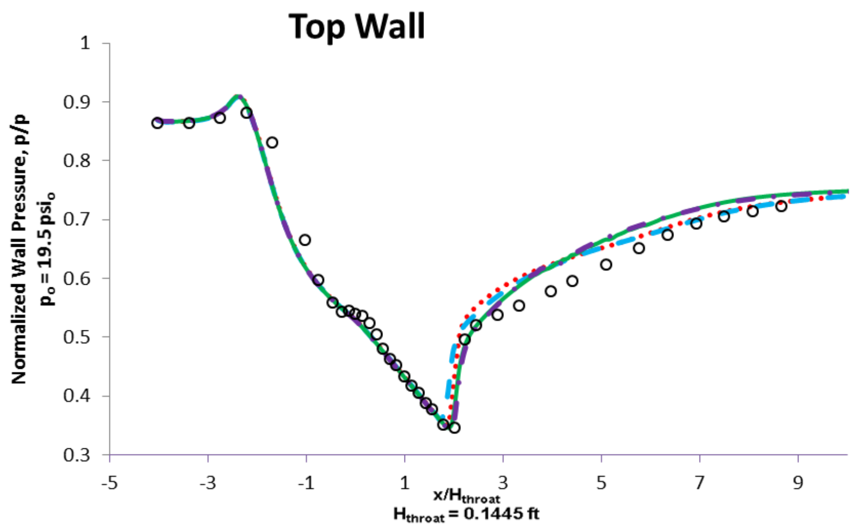




Test Cases
Sajben Diffuser – Strong Shock Case

FUN3D RSM Results
Wall Pressure





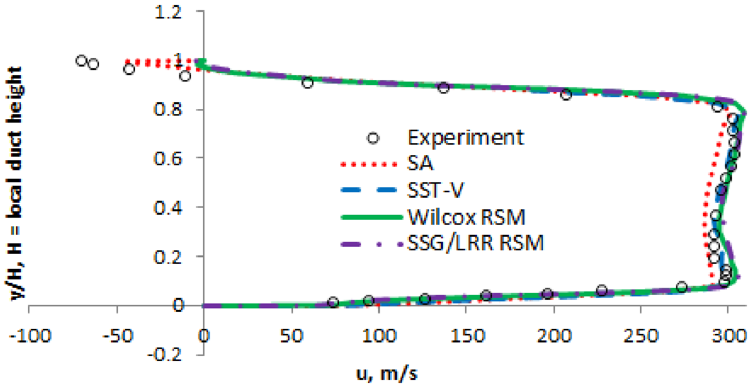
Test Cases

Sajben Diffuser – Strong Shock Case

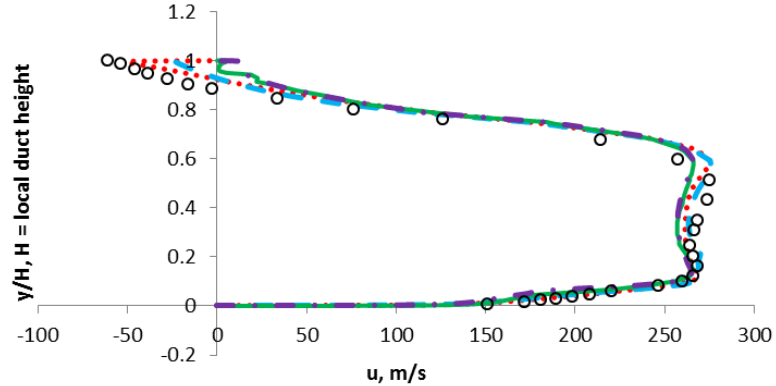
FUN3D RSM Results

Velocity

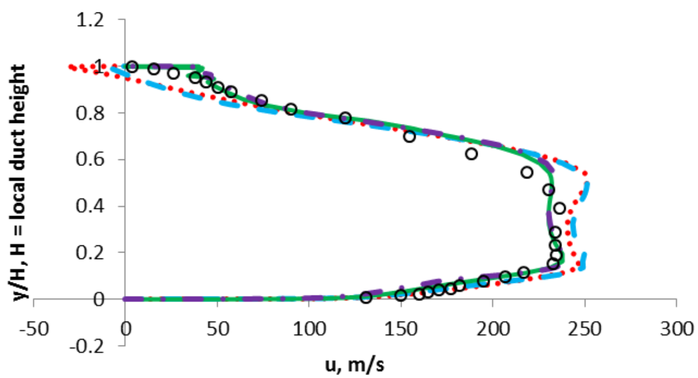
x/H = 2.9



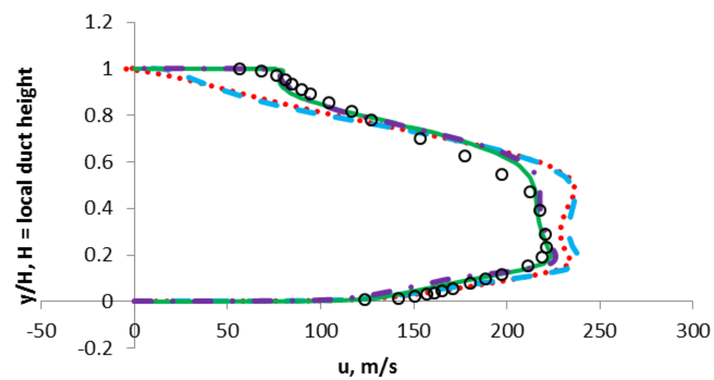
x/H = 4.6



x/H=6.4



x/H=7.5

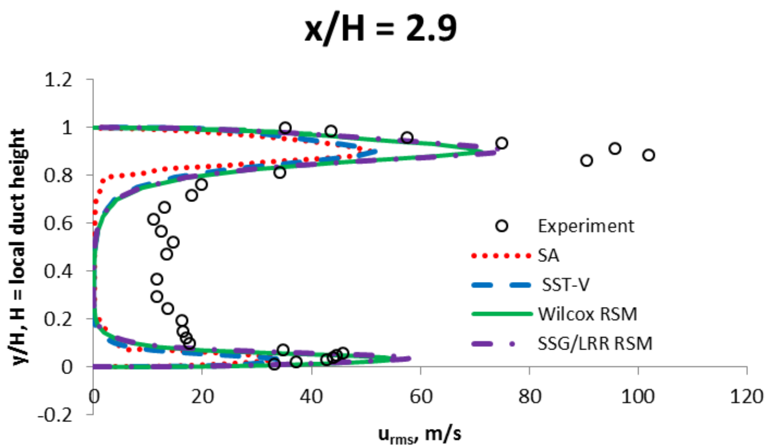


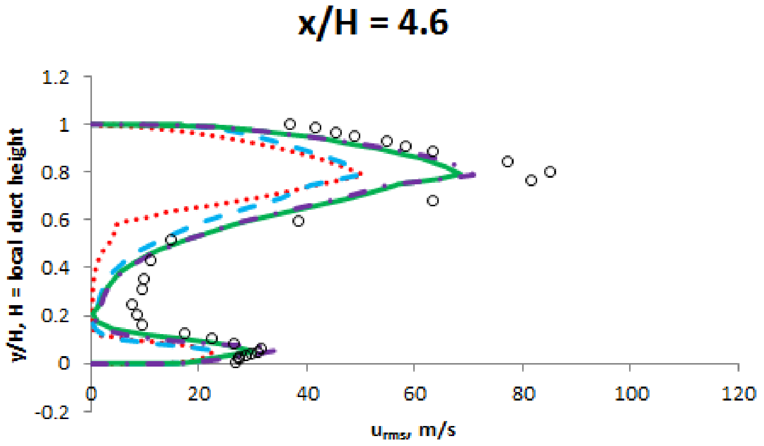
Test Cases

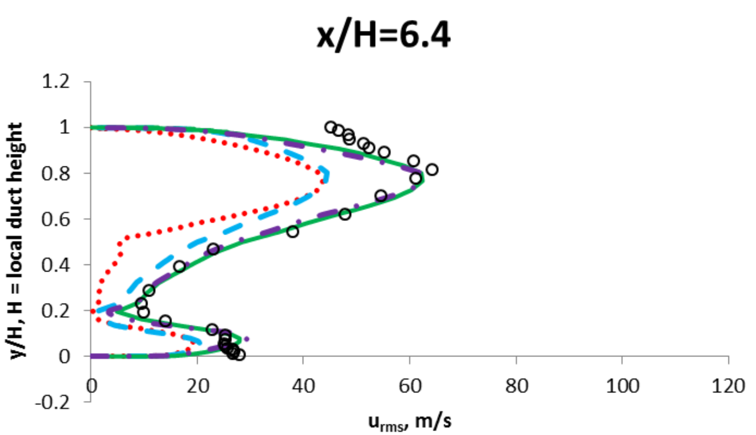
Sajben Diffuser – Strong Shock Case

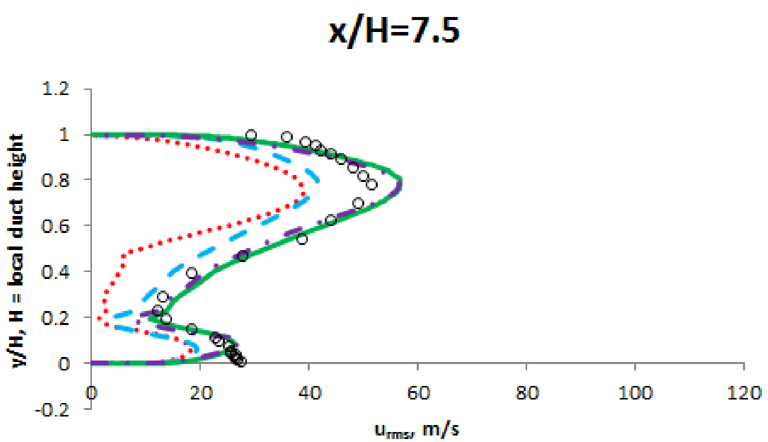
FUN3D RSM Results

Axial Turbulence Intensity









Test Cases

Sajben Diffuser – Strong Shock Case

Summary of Results

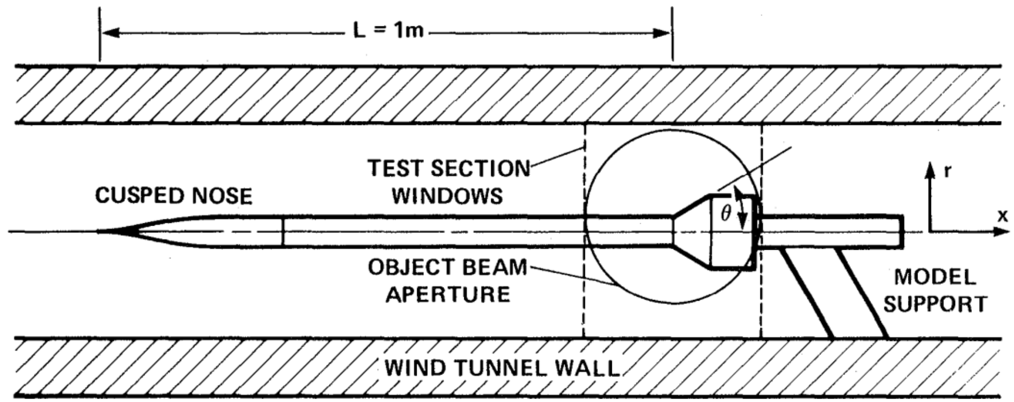
- The two stress-omega models give very similar results.
- Axial turbulence intensity profiles show better agreement with experiment than the SA and SST models.
- The velocity profiles show that the SA model does the best job of predicting the separation, however the stress-omega models are better at predicting the velocity profiles in the downstream portion of the duct.

Test Cases

30° Axisymmetric Compression Corner

Experiment

- J. Brown et al, NASA Ames
- Mach 2.85,   Re = 16 x 10^6/m
- Data
  - LDV
    - Mean velocities
    - Reynolds stresses
  - Surface static pressures
  - Interferometry
  - Schlieren
  - Oil flow



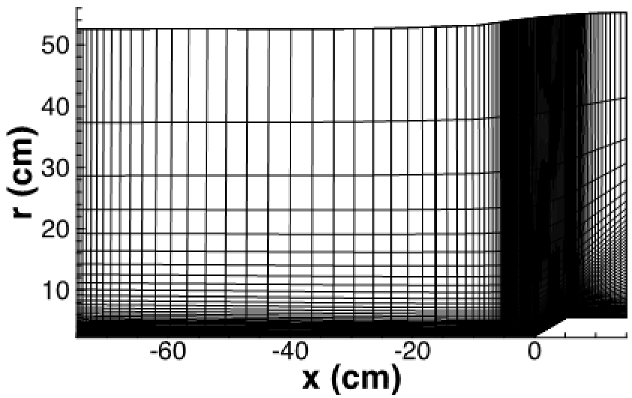
- Dunagan, S.E., Brown, J.L. and Miles, J.B. ,” Interferometric Data for a Shock/Wave Boundary-Layer Interaction,” NASA TM 88227, Sept. 1986.
- Brown, J.D., Brown, J.L. and Kussoy, M.I., “A Documentations of Two- and Three-Dimensional Shock-Separated Turbulent Boundary Layers,” NASA TM 101008, July, 1988.
- *Settles, G.S., and Dodson, L.J., “Hypersonic Shock/Boundary-Layer Interaction Database NASA CR 177577, April 1991
- Wideman, J., Brown, J., Miles, J., and Ozcan, O., “Surface Documentation of a 3-D Supersonic Shock-Wave/Boundary-Layer Interaction,” NASA TM 108824, 1994

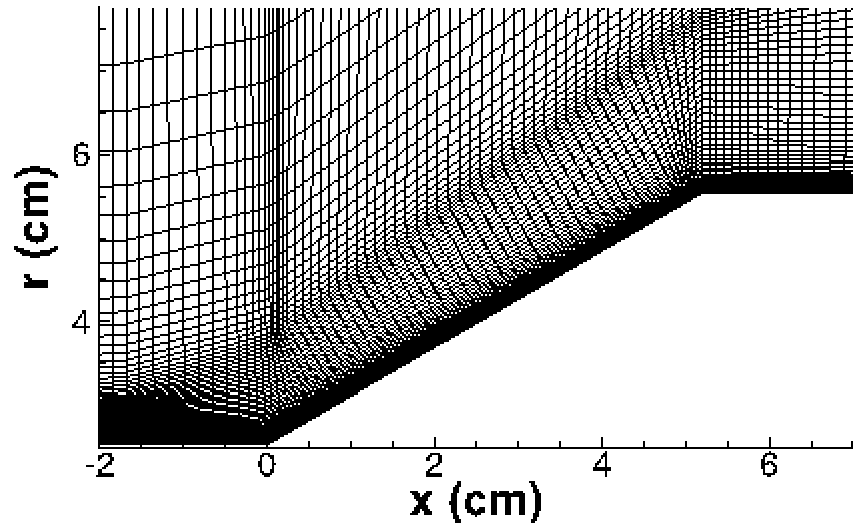
*primary data source

Test Cases

30° Axisymmetric Compression Corner

Grid and Flow Features

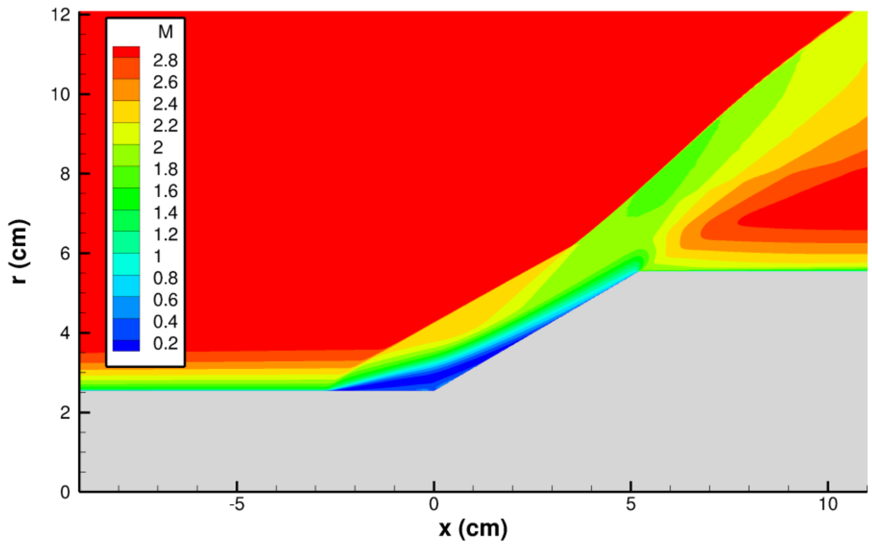




<u>Grid</u>

- 1265 axial points, 729 radial points
- SA, SST-V single-cell axisymmetric wedge grid (922,185 points)
- RSMs 90-degree, 17 circumferential points (15,478,857 points)
- Orthogonal to the wall, y+=0.2
- Axial lines parallel to shock

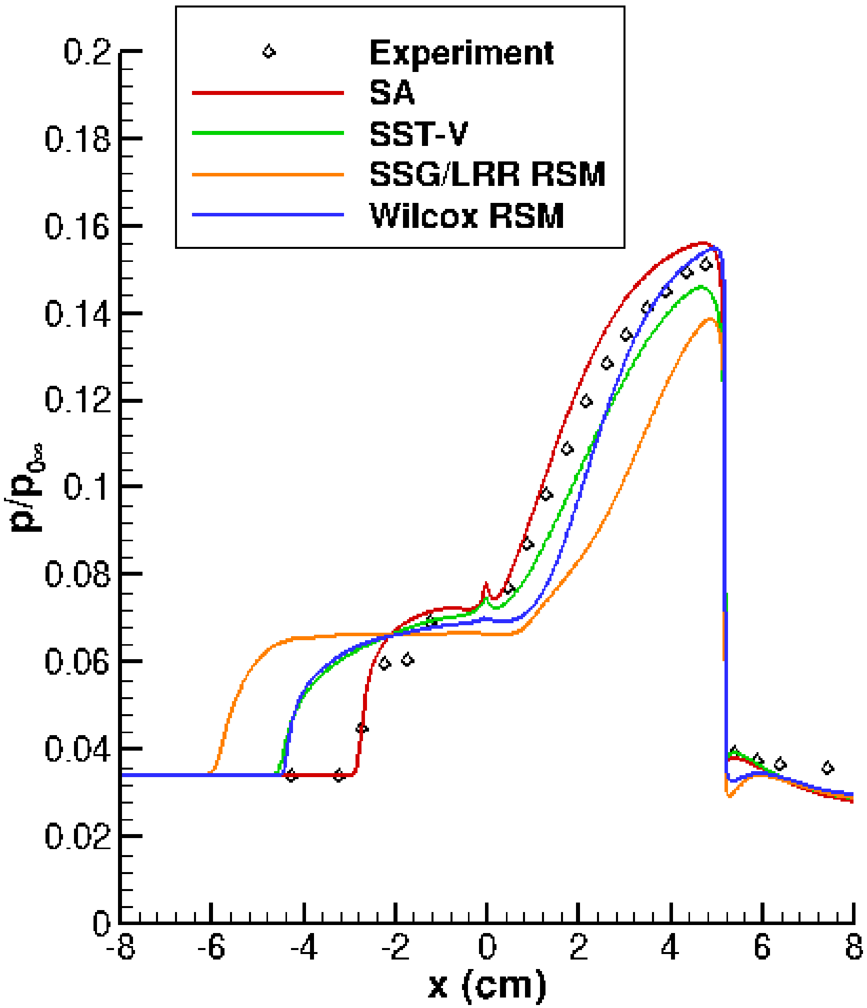
Mach Contours



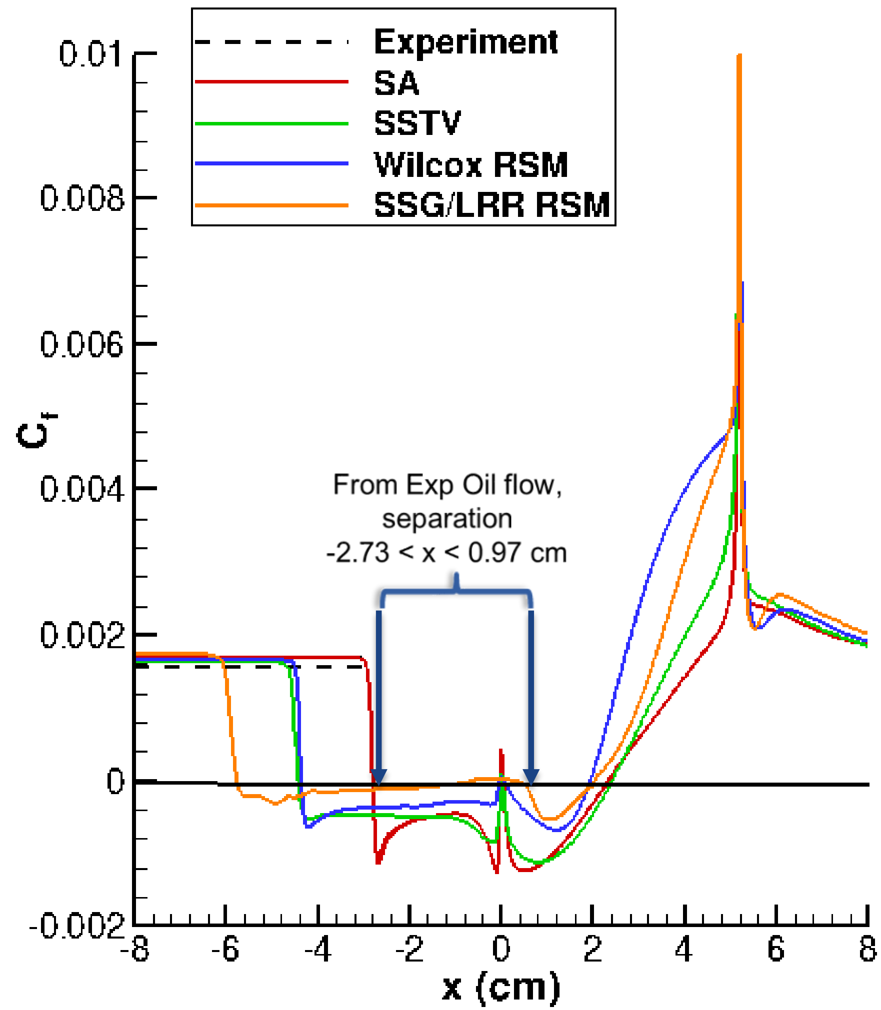
Test Cases

30° Axisymmetric Compression Corner

Pressure



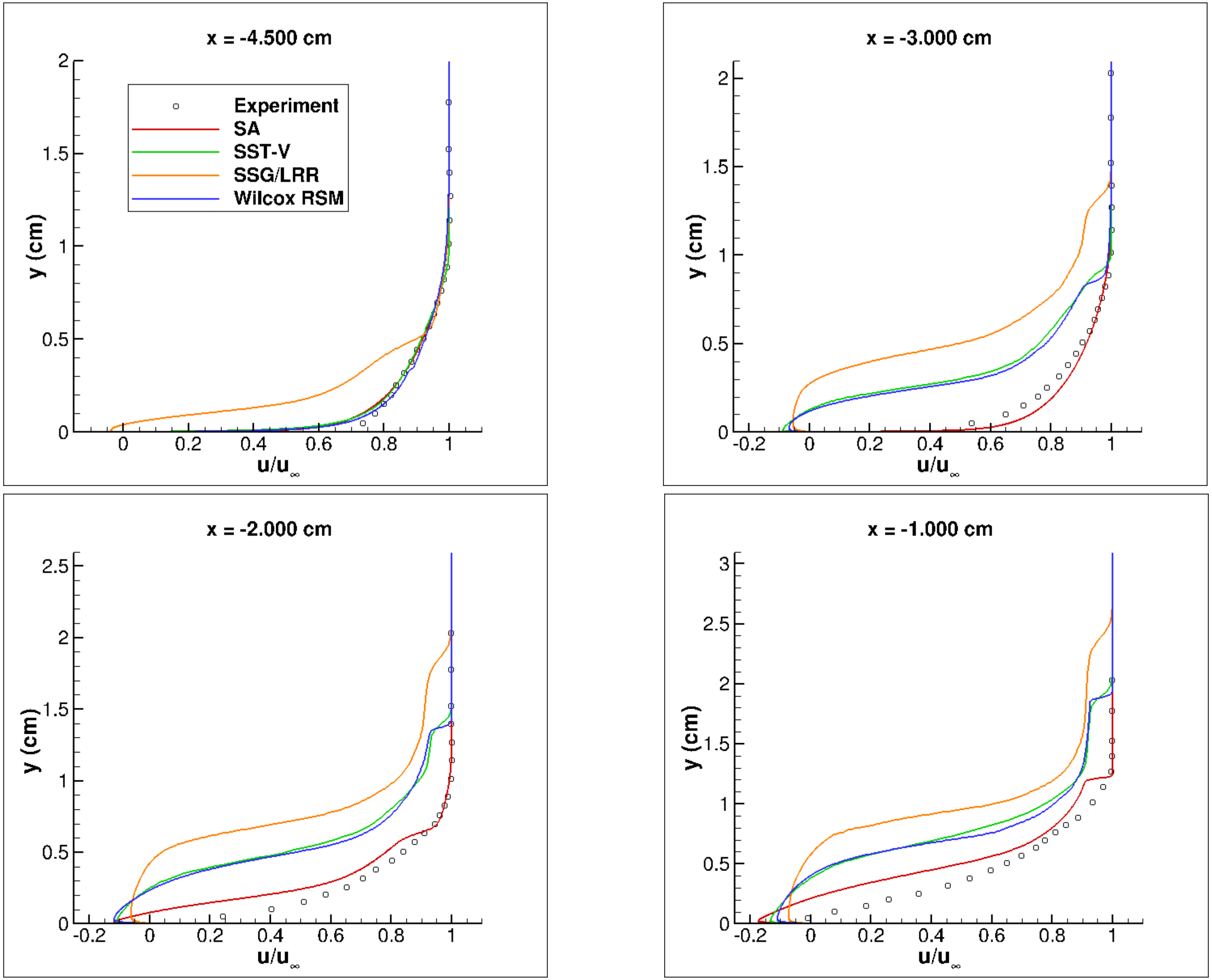
Skin Friction



Test Cases

30° Axisymmetric Compression Corner

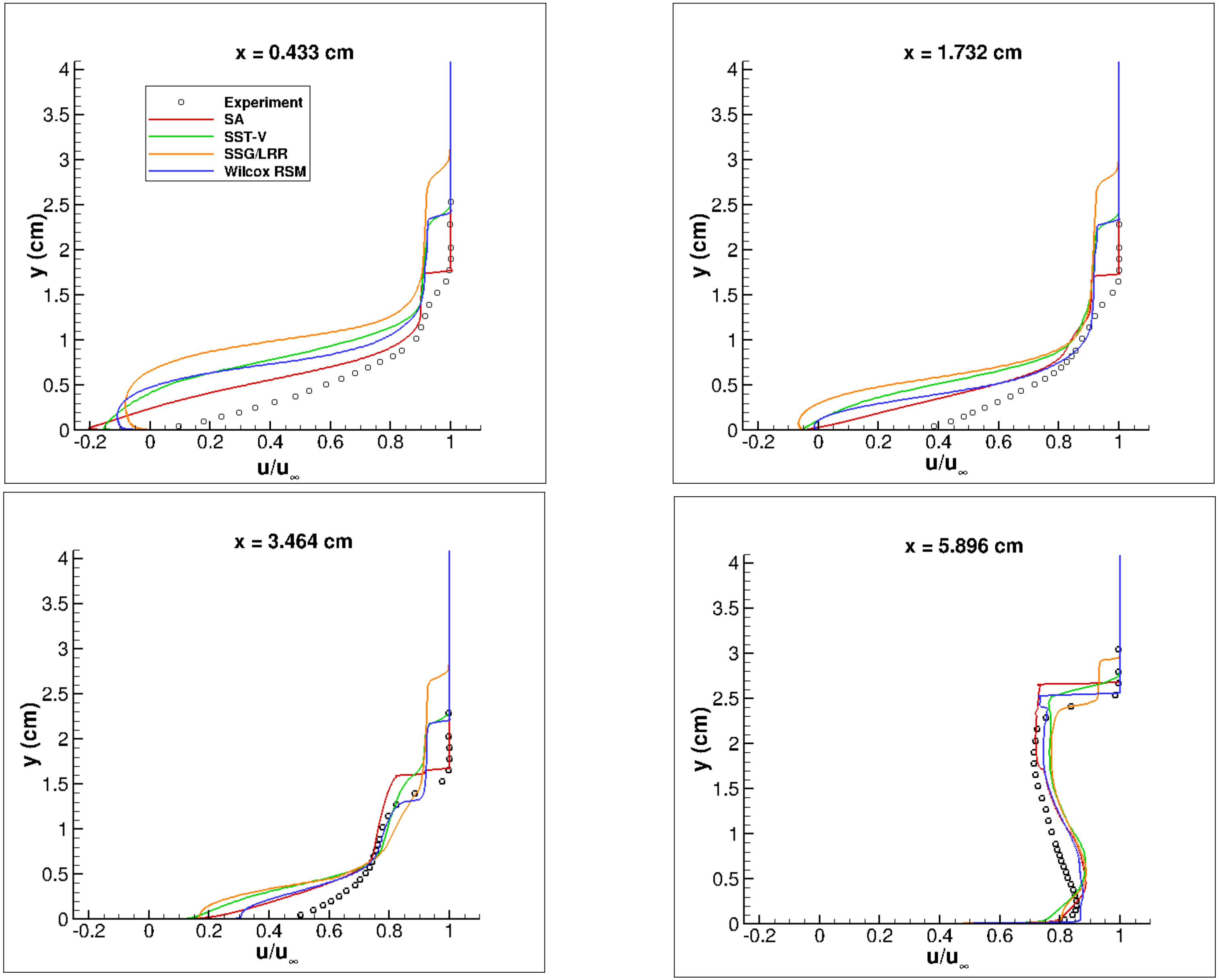
Velocity Profiles – Upstream of Flare



Test Cases

30° Axisymmetric Compression Corner

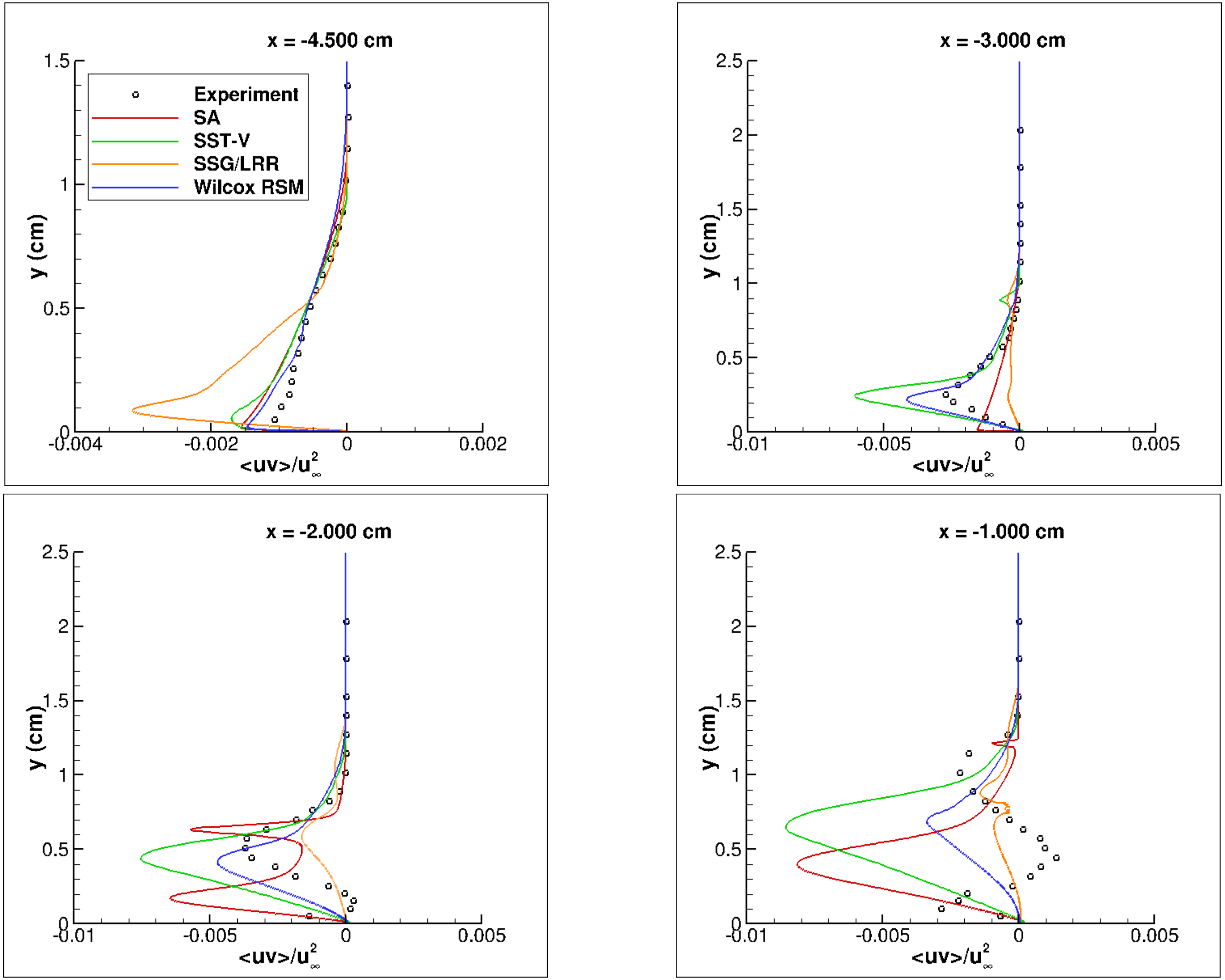
Velocity Profiles – Downstream of Flare Corner



Test Cases

30° Axisymmetric Compression Corner

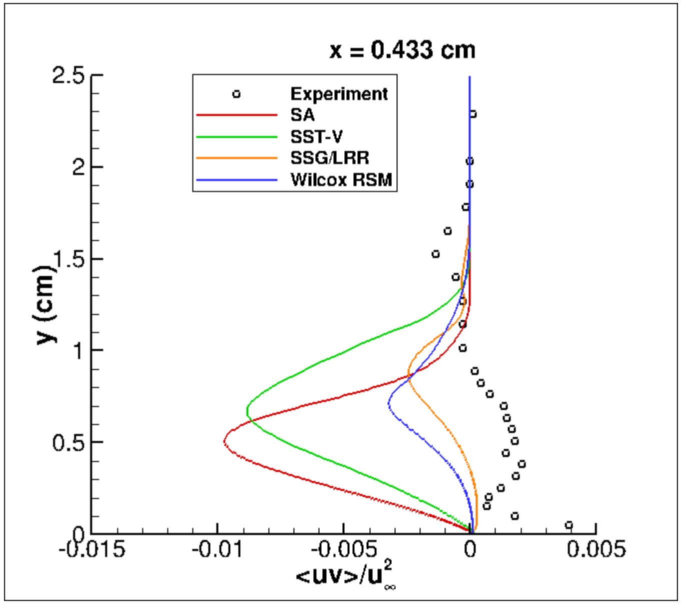
Turbulent Shear Stress – Upstream of Flare

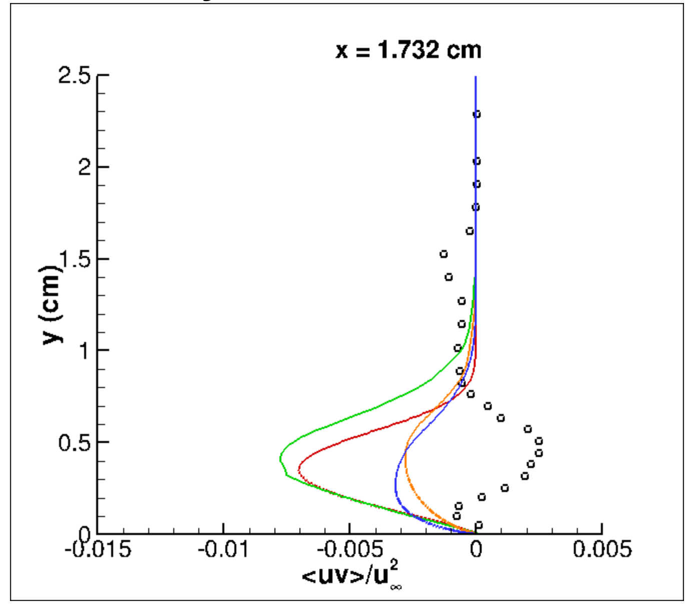


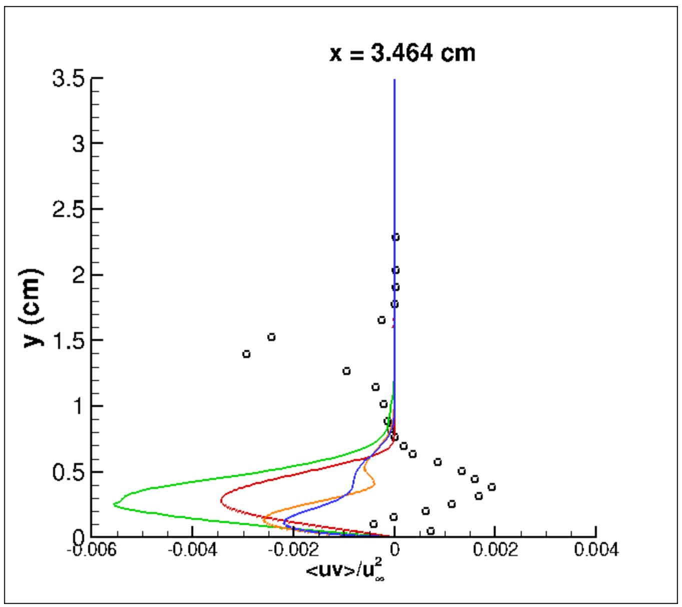
Test Cases

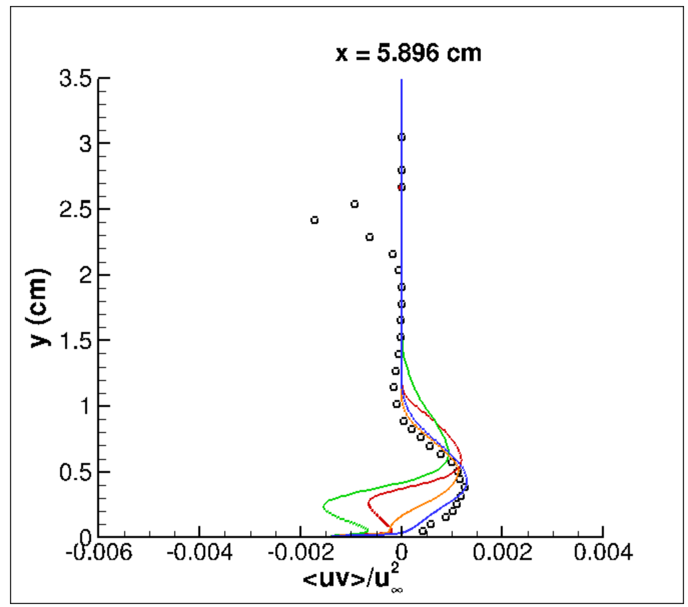
30° Axisymmetric Compression Corner

Turbulent Shear Stress – Downstream of Flare Corner









Test Cases
30° Axisymmetric Compression Corner
Summary of Results

- The Wilcox and SSG-LRR RSMs behaved quite differently.

- The Wilcox RSM and the SST-V model have similar behavior

- The Wilcox RSM predicted the correct pressure rise on the compression surface, whereas the SSG-LRR RSM significantly under-predicted the pressure rise.

- The SA model did the best job of predicting the separation location and the pressure rise. It also did the best job at predicting the velocity profiles.

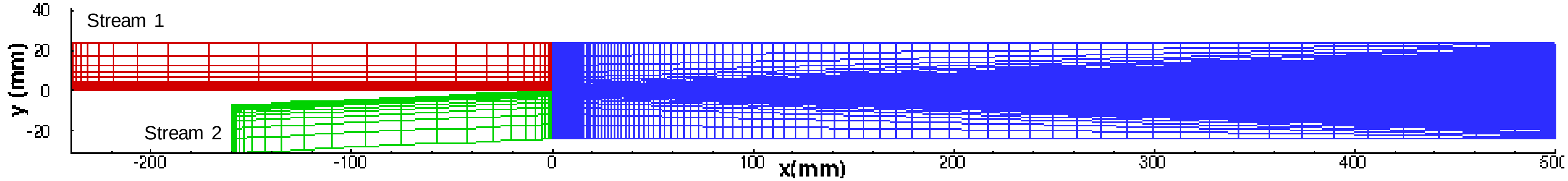
- The Wilcox RSM may have an advantage at predicting the shear stress profiles.

- **Conclusion**

  While the Wilcox RSM may offer some slight benefits in predicting the shear stress profiles for this case. The SA model gave the best results overall. The SSG-LRR RSM performed poorly.

Test Cases
Compressible Mixing Layer
Experiment

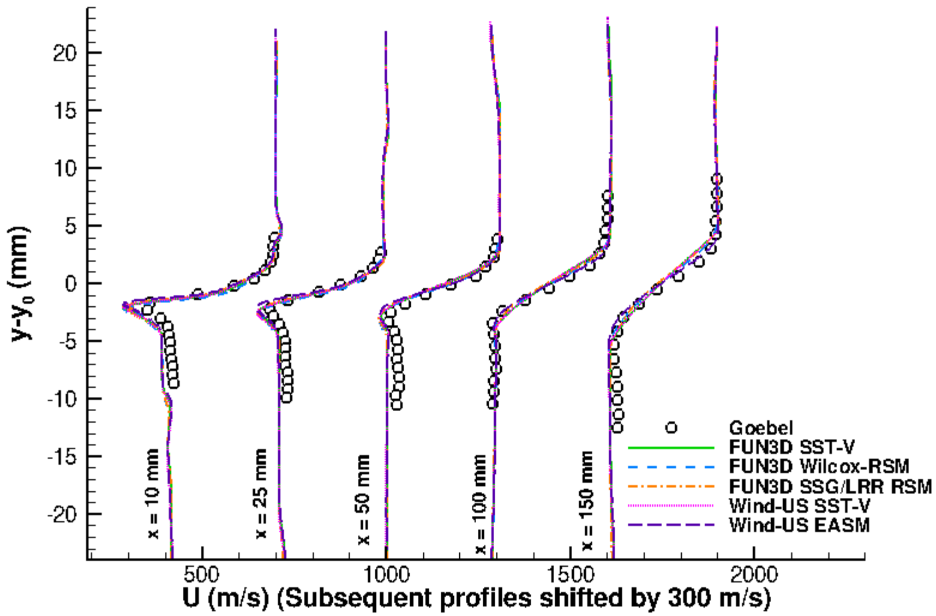
- Goebel, Dutton, & Gruber- Univ.of Illinois (1991)
- Test Case 2, Convective Mach No., $M_c = 0.46$, $Re = 12 \times 10^6$/m
- Data available:
  - LDV Mean velocities and Reynolds Stress
  - Growth Rates
  - Schlieren

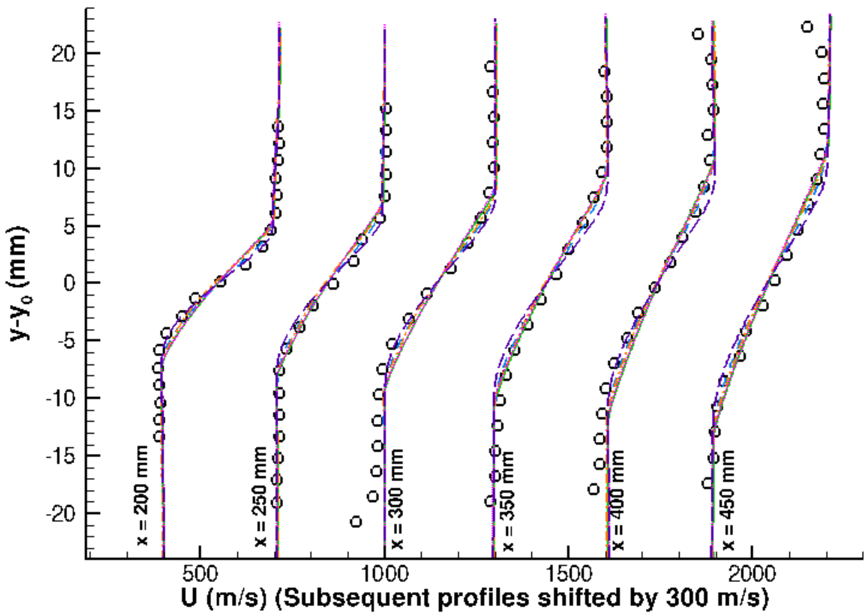


	Primary (Stream 1)	Secondary (Stream 2)
Mach	1.91	1.36
P(kpa)	49	49
T(K)	334	215
U(m/s)	700	399
a(m/s)	366	293
ρ(kg/m^3)	0.51	0.79

- Goebel, S.G. and Dutton, J.C., "Experimental Study of Compressible Turbulent Mixing Layers," AIAA Journal, vol. 29, no. 4, pp. 538-546, April, 1991.
- Goebel, S.G. "An Experimental Investigation of Compressible Turbulent Mixing Layers," Ph.D. Thesis, Dept. of Mech. and Ind. Eng., Univ. of Illinois., Urbana, Ill., 1990.
-Gruber, M.R. and Dutton, J.C., "Three-Dimensional Velocity Measurements in a Turbulent Compressible Mixing Layer," AIAA Paper 92-3544, July 1992

Test Cases
Compressible Mixing Layer
Mean Velocity

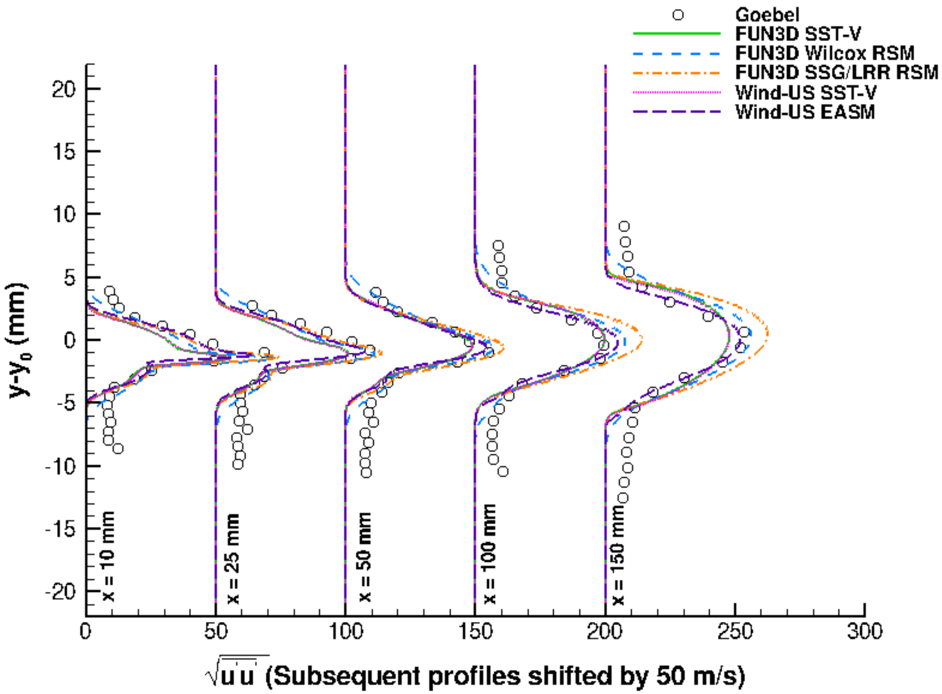


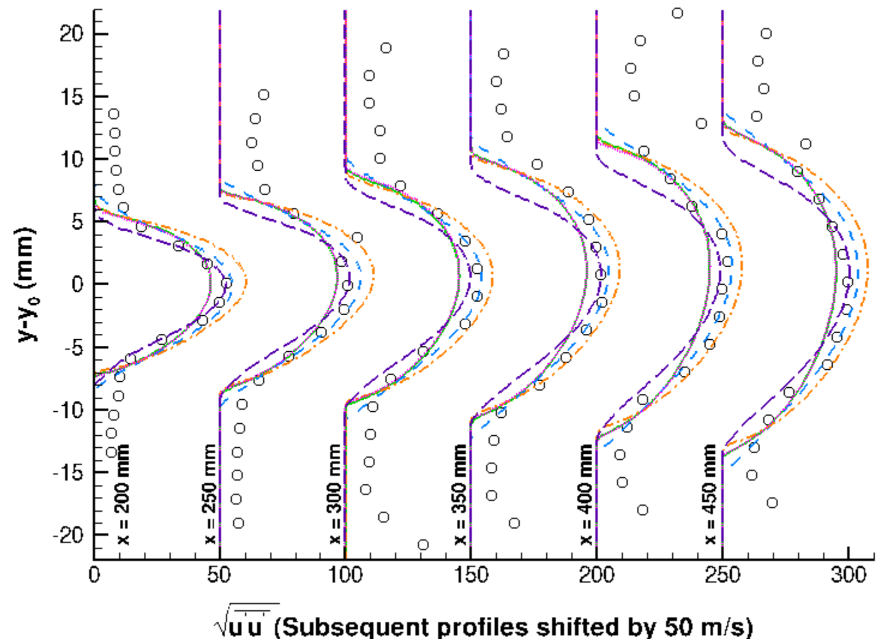


Test Cases

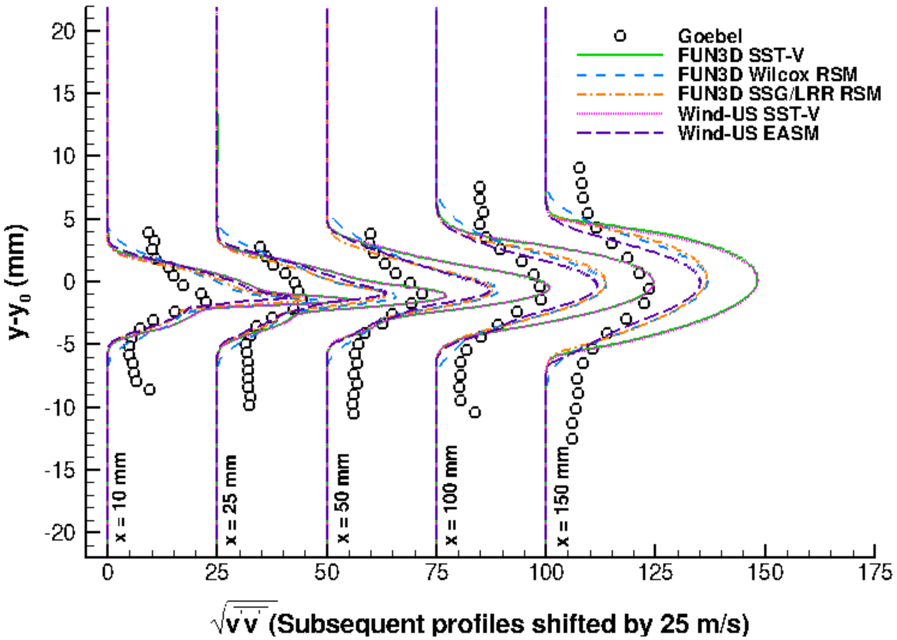
Compressible Mixing Layer

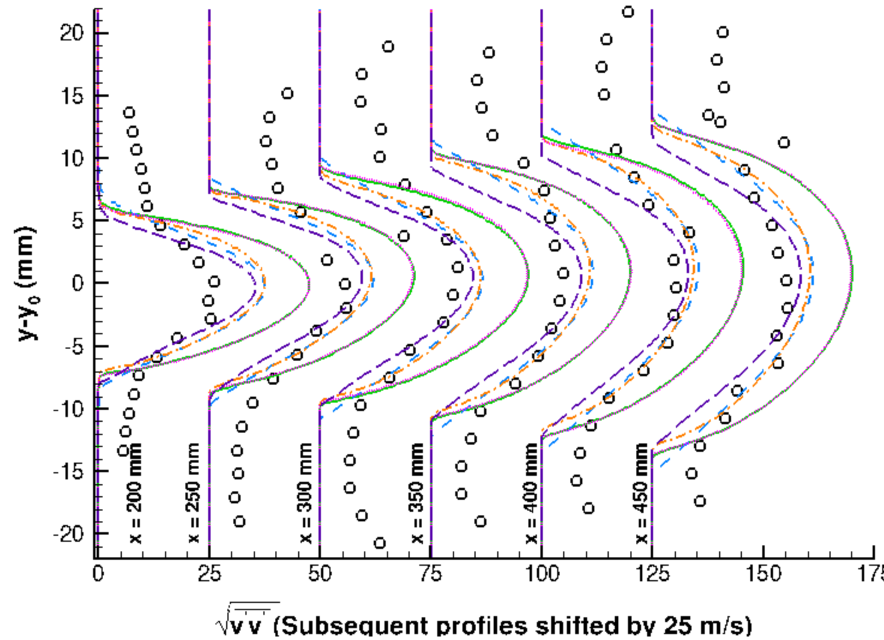
Streamwise Turbulence Intensity





Test Cases
Compressible Mixing Layer
Transverse Turbulence Intensity

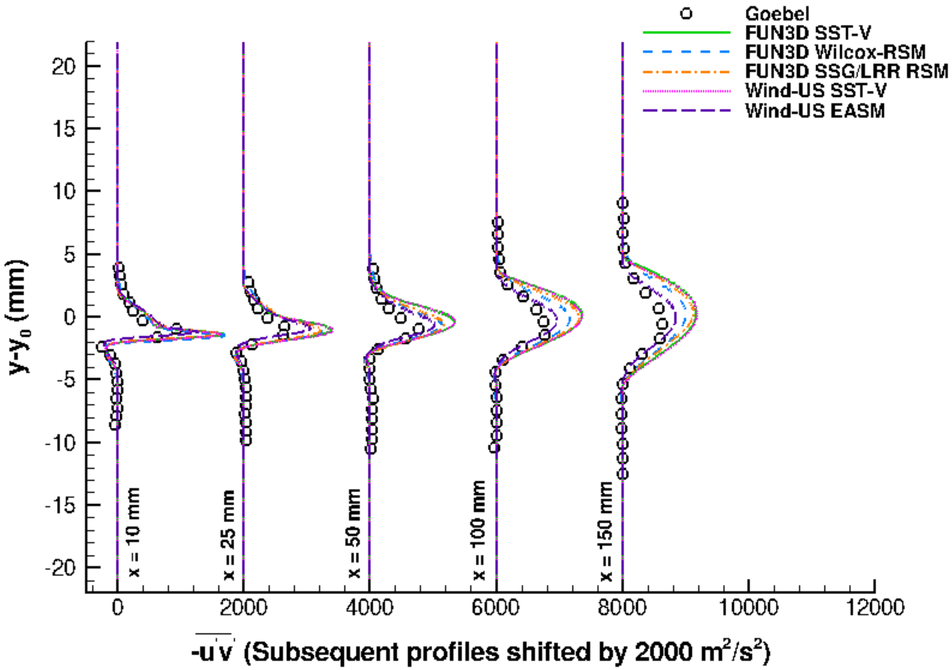


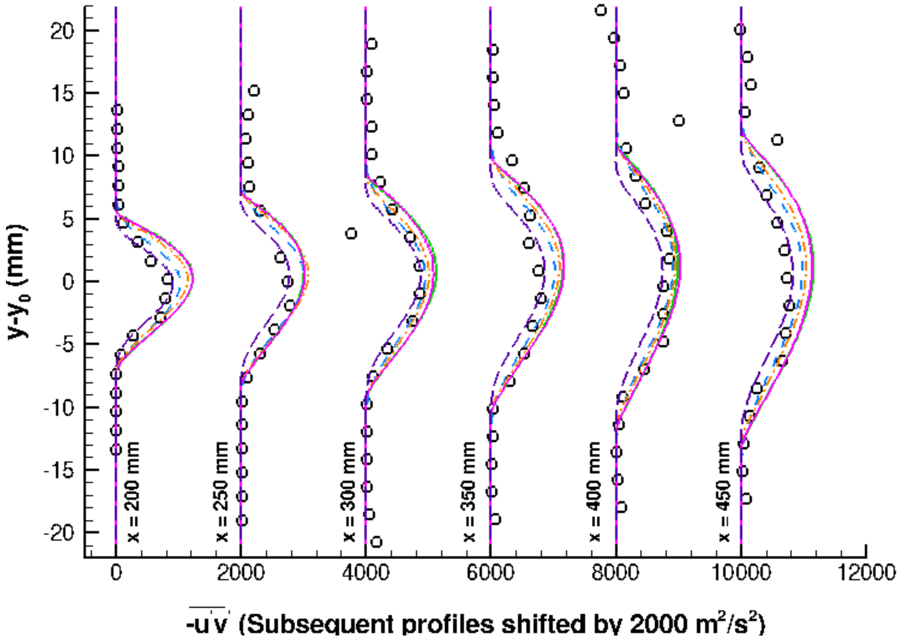


Test Cases

Compressible Mixing Layer

Turbulent Shear Stress





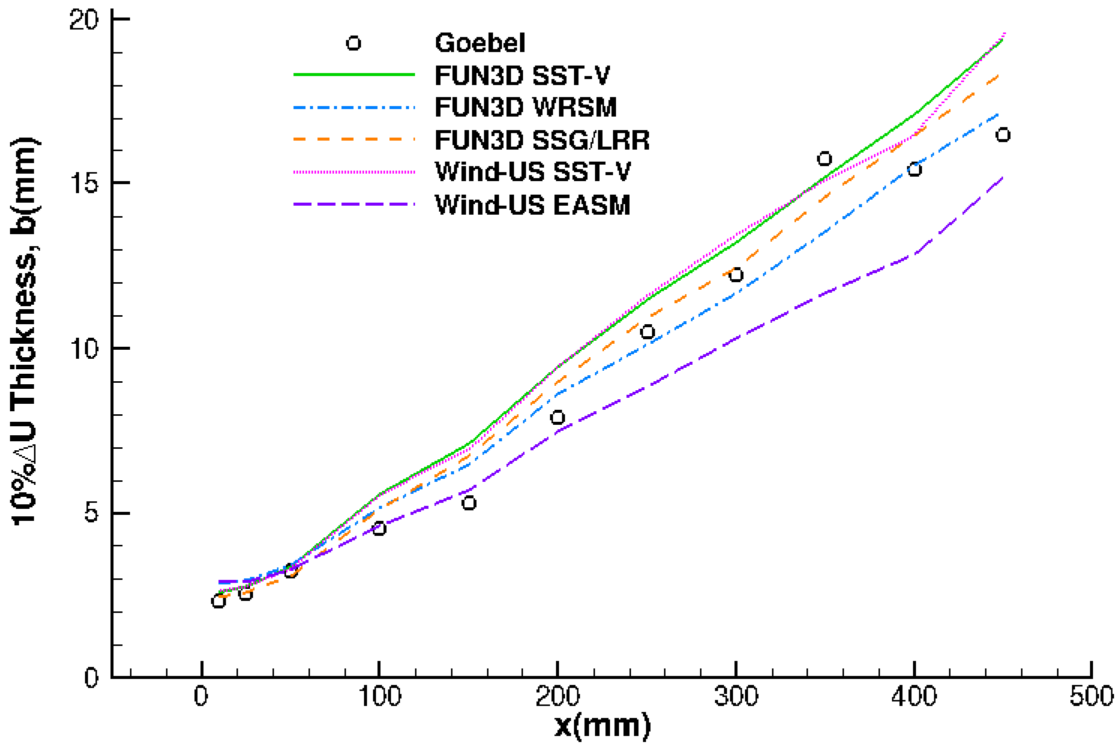
Test Cases

Compressible Mixing Layer

Shear Layer Thickness

Shear layer thickness definition:
The distance, *b*, between transverse locations where:

$$\tilde{u} = \tilde{u}_1 - 0.1\Delta\tilde{u} \text{ and } \tilde{u} = \tilde{u}_2 + 0.1\Delta\tilde{u}.$$



Test Cases
Compressible Mixing Layer
Summary of Results

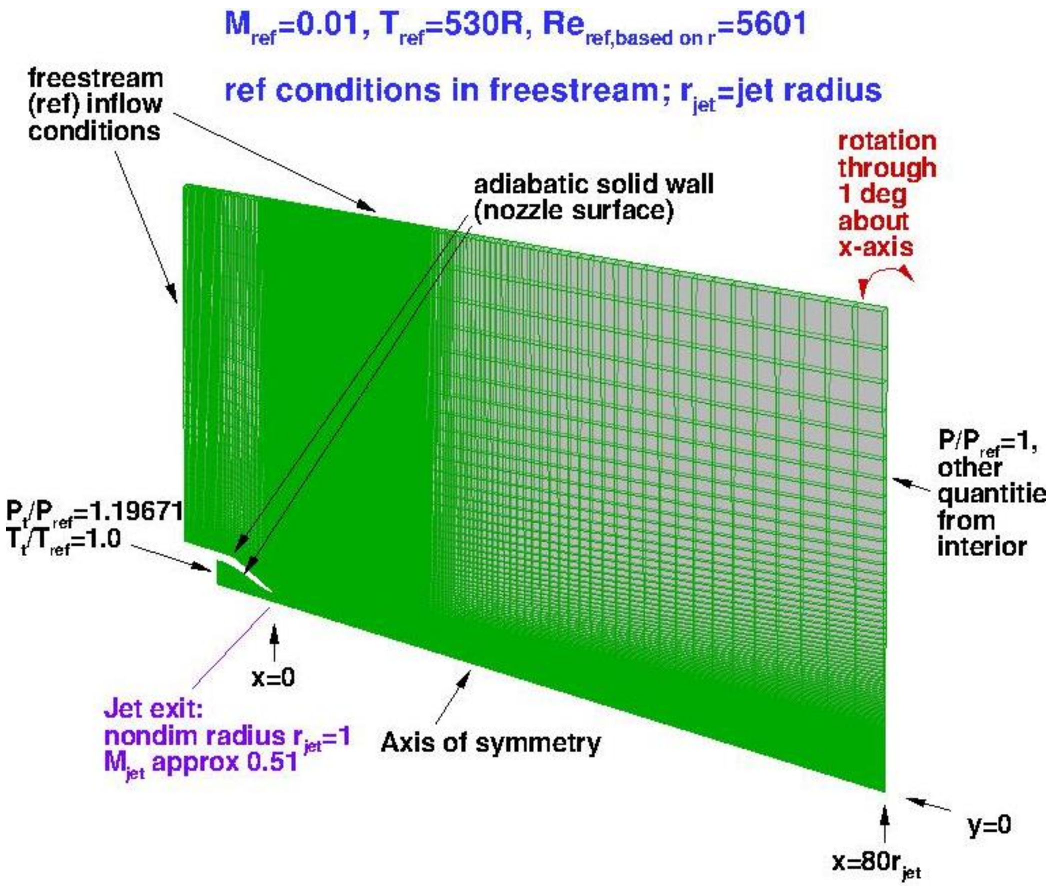
- Results using FUN3D with the SST-V model agree well with the Wind-US SST-V results.
- All of the models compute the velocity profiles in the mixing layer well.
- The Wilcox and SSG/LRR RSM and the EASM turbulence models are better than the SST-V model at predicting the turbulence quantities u'u', v'v' and u'v'.
- The Wilcox and SSG/LRR RSM models give very similar results for v'v' and u'v'. For u'u', the Wilcox RSM model does slightly better.

SIGNIFICANCE

The Wilcox and the SSG/LRR full Reynolds stress turbulence models give improved turbulence predictions over the SST-V two equation turbulence model for this supersonic mixing layer case.

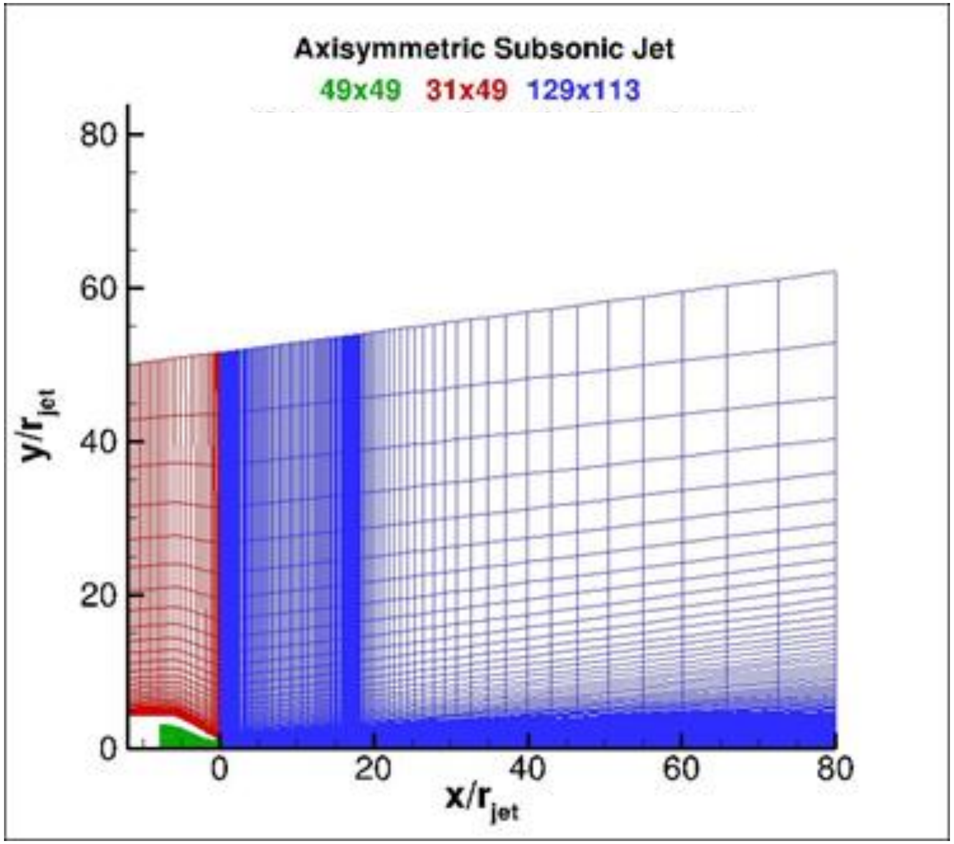
Test Cases
Axisymmetric Subsonic Jet
Experiment

- Bridges and Wernet
- ARN2, $D_{jet} = 2$ in
- $M_{jet} = \frac{u_{jet}}{a_{jet}} = 0.51$
- PIV data



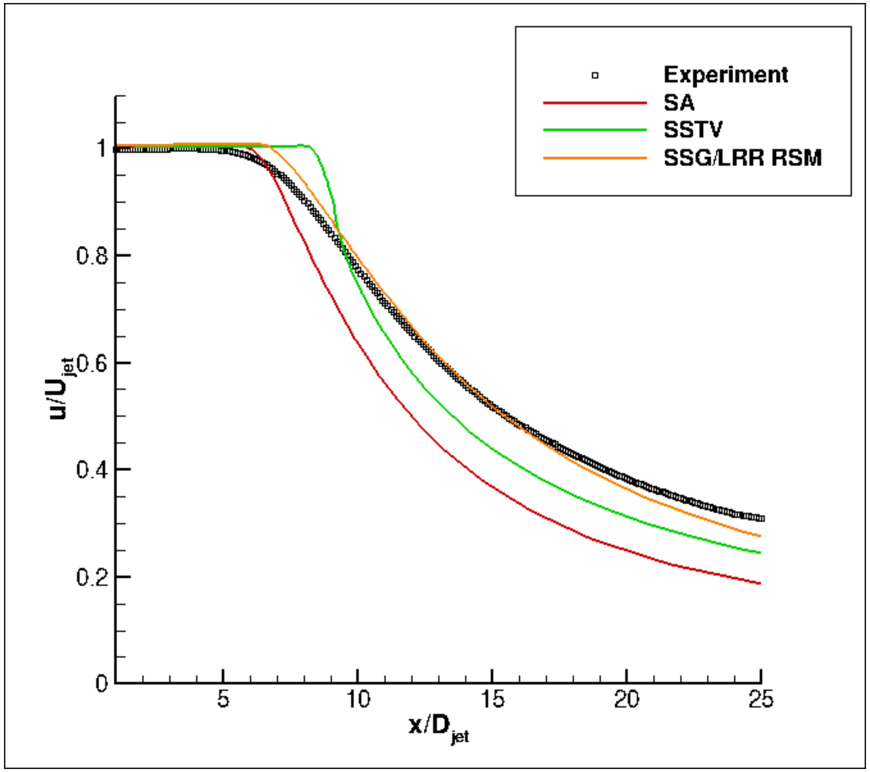
- Bridges, J. and Wernet, M. P., "Establishing Consensus Turbulence Statistics for Hot Subsonic Jets," AIAA Paper 2010-3751, 16th AIAA/CEAS Aeroacoustics Conference, Stockholm, Sweden, June 2010.
- Bridges, J. and Wernet, M. P., "The NASA Subsonic Jet Particle Image Velocimetry (PIV) Dataset," NASA/TM-2011-216807, November 2011.

Test Cases
Axisymmetric Subsonic Jet
Grids – From TMR (turbmodels.larc.nasa.gov)

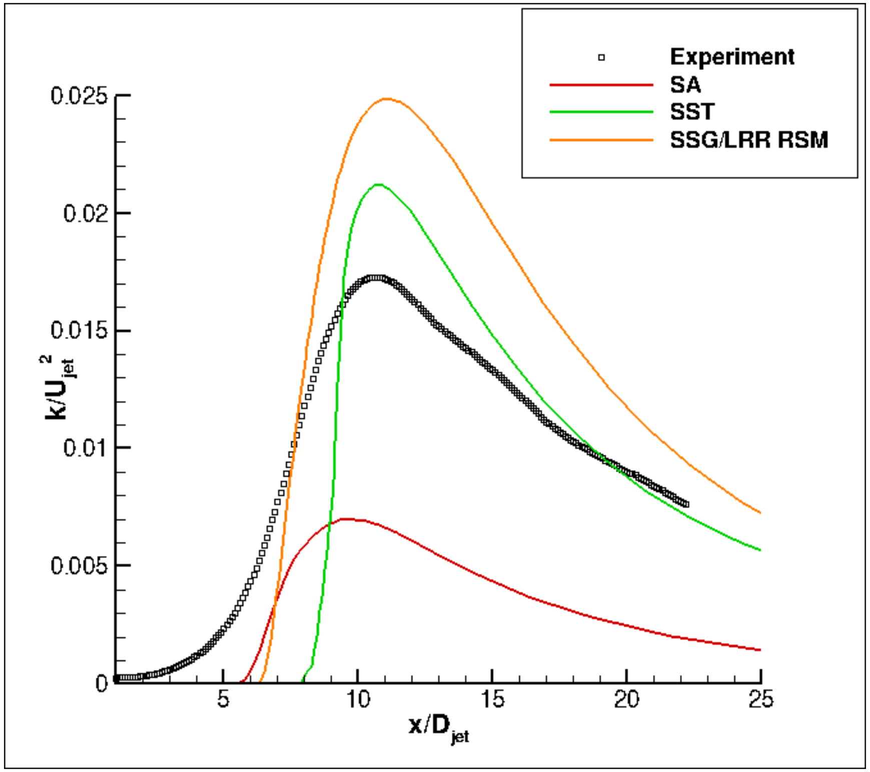


Test Cases
Axisymmetric Subsonic Jet
Centerline Profiles

Axial Velocity

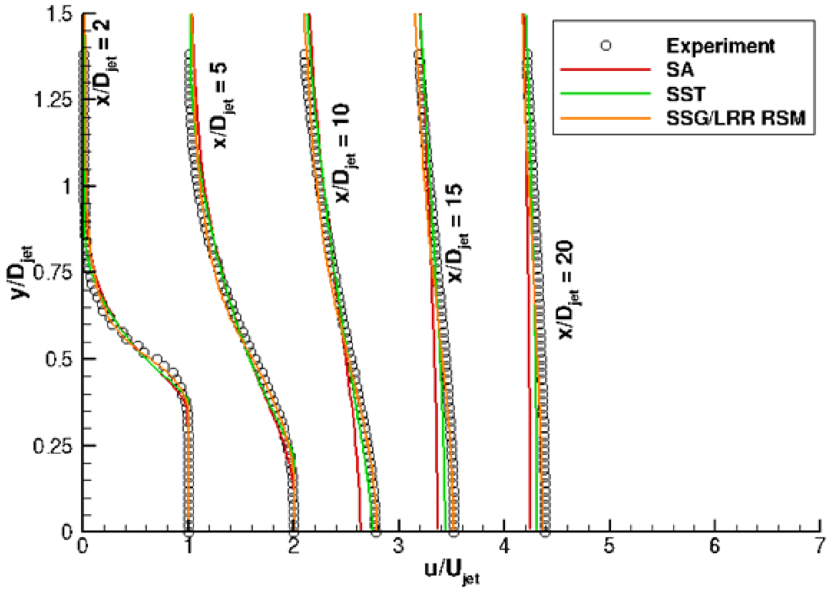


Turbulent Kinetic Energy



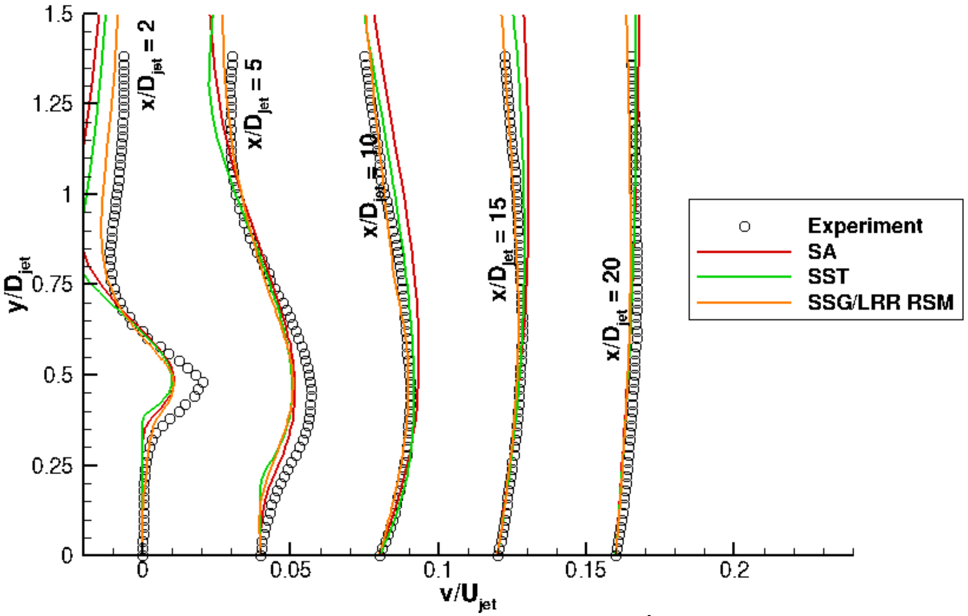
Test Cases
Axisymmetric Subsonic Jet
Radial Profiles

x-Velocity



(Subsequent profiles shifted by $u/U_{jet} = 1.0$)

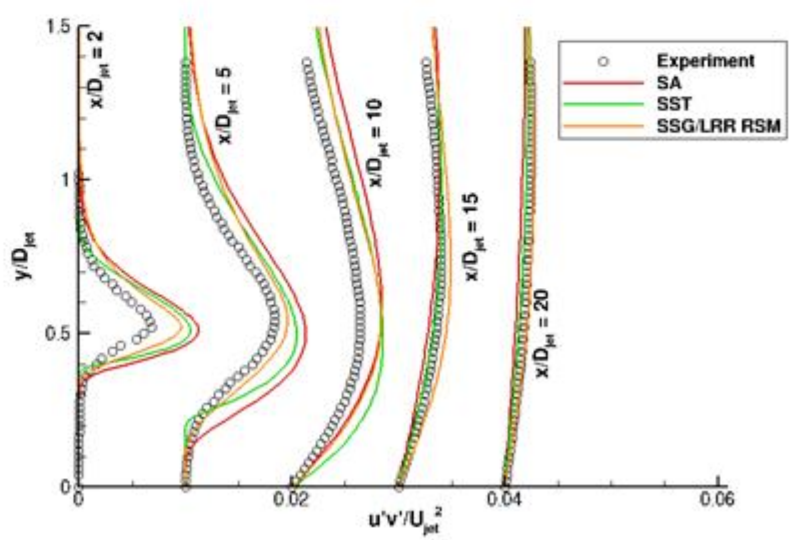
y-Velocity



(Subsequent profiles shifted by $v/U_{jet} = 0.04$)

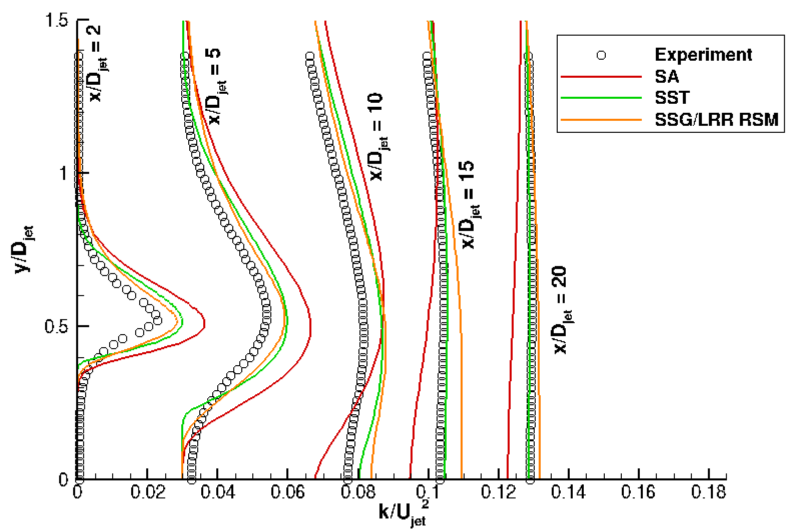
Test Cases
Axisymmetric Subsonic Jet
Radial Profiles

Turbulent Shear Stress



(Subsequent profiles shifted by $u'v'/U_{jet}^2 = 0.01$)

Turbulent Kinetic Energy



(Subsequent profiles shifted by $k/U_{jet}^2 = 0.03$)

Test Cases
Axisymmetric Subsonic Jet
Summary of Results

- The SSG/LRR model shows some benefits over the SA and SST-V models at predicting the mixing.

Summary

- Two RSMs available in the FUN3D code, the Wilcox and the SSG/LRR, were evaluated for four test cases: a transonic diffuser, a supersonic axisymmetric compression corner, a supersonic compressible planar mixing layer, and a subsonic axisymmetric jet.

- RSM results were compared with solutions computed using the SA and SST-V turbulence models, and an EASM (planar mixing layer).

- Transonic diffuser - results were somewhat inconclusive as to the benefits of the RSMs

- The supersonic axisymmetric compression corner – the SA model was best for computing the pressure rise and the separation location and length. The Wilcox RSM gave results similar to SST-V, and the SSG/LRR RSM severely over-predicted the onset of separation. All models had difficulty computing the boundary layer profiles and turbulence quantities in the separated region, and no additional benefit was gained by using RSMs.

- Supersonic planar mixing layer – the RSMs gave the best predictions of the turbulence intensity, turbulent shear stress and shear layer thickness

- Subsonic axisymmetric jet – SSG/LRR predicted the mixing of the core velocity the best

Conclusions

- The four cases examined are flows that are challenging for current turbulence models because they contain mixing, shock waves and/or separation.
- Overall, the RSMs showed benefit over the SA and SST-V models for the planar mixing layer and the axisymmetric jet flow, and may be useful for future nozzle calculations.
- While the cases examined are challenging flows, they are still relatively simple in geometry and flow features.
- More complex flow cases may reveal more benefits of the RSMs and are recommended for future study.

Questions?

Extra Slides

Turbulence Models, cont'd

- Seven-equation omega-based full Reynolds Stress models:
  - SSG/LRR RSM:

$$\frac{\partial(\bar{\rho}\tau_{ij})}{\partial t} + \frac{\partial\left(\bar{\rho}\tau_{ij}\tilde{u}_k\right)}{\partial x_k} = -\bar{\rho}P_{ij} - \bar{\rho}\Pi_{ij} + \bar{\rho}\varepsilon_{ij} - \bar{\rho}D_{ij} - \bar{\rho}\mathcal{M}_{ij}$$

$$\frac{\partial(\bar{\rho}\omega)}{\partial t} + \frac{\partial(\bar{\rho}\omega\tilde{u}_k)}{\partial x_k} = \alpha_\omega \frac{\omega}{\tilde{k}} \frac{\bar{\rho}P_{kk}}{2} - \beta_\omega \bar{\rho}\omega^2 + \frac{\partial}{\partial x_k}\left[\left(\bar{\mu} + \sigma_\omega \frac{\bar{\rho}\tilde{k}}{\omega}\right)\frac{\partial\omega}{\partial x_k}\right] + \sigma_d \frac{\bar{\rho}}{\omega} \max\left(\frac{\partial\tilde{k}}{\partial x_k}\frac{\partial\omega}{\partial x_k}, 0\right)$$

- Blended Speziale-Sarkar-Gatski/Launder-Reece-Rodi pressure strain model

$$\Pi_{ij} = -\left(C_1\varepsilon + \frac{1}{2}C_1^* P_{kk}\right)\tilde{a}_{ij} + C_2\varepsilon\left(\tilde{a}_{ik}\tilde{a}_{kj} - \frac{1}{3}\tilde{a}_{kl}\tilde{a}_{kl}\delta_{ij}\right) + \left(C_3 - C_3^*\sqrt{\tilde{a}_{kl}\tilde{a}_{kl}}\right)\tilde{k}\tilde{S}_{ij}^*$$
$$+ C_4\tilde{k}\left(\tilde{a}_{ik}\tilde{S}_{jk} + \tilde{a}_{jk}\tilde{S}_{ik} - \frac{2}{3}\tilde{a}_{kl}\tilde{S}_{kl}\delta_{ij}\right) + C_5\tilde{k}\left(\tilde{a}_{ik}\tilde{W}_{jk} + \tilde{a}_{jk}\tilde{W}_{ik}\right)$$

$$P_{ij} = \tau_{ik}\frac{\partial\tilde{u}_j}{\partial x_k} + \tau_{jk}\frac{\partial\tilde{u}_i}{\partial x_k}$$ Production

$$\bar{\rho}\varepsilon_{ij} = \frac{2}{3}\bar{\rho}\delta_{ij}\varepsilon$$ Dissipation

Turbulence Models, cont'd

SSG/LRR RSM

$$\tilde{a}_{ij} = -\frac{\tau_{ij}}{\tilde{k}} - \frac{2}{3}\delta_{ij}$$

Anisotropy

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

Strain Rate Tensor

$$\tilde{S}_{ij}^{*} = \tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}$$

Traceless Strain Rate Tensor

$$\widetilde{W}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

Averaged Rotation Tensor

$$\overline{\rho}D_{ij} = -\frac{\partial}{\partial x_k}\left[\left(\bar{\mu}\delta_{kl} - D\frac{\overline{\rho}\tau_{kl}\tilde{k}}{\varepsilon}\right)\frac{\partial(\tau_{ij})}{\partial x_l}\right]$$

Generalized Diffusion

Simple diffusion model:

$$\overline{\rho}D_{ij} = -\frac{\partial}{\partial x_k}\left[\left(\bar{\mu} - \frac{D}{C_\mu}\mu_T\right)\frac{\partial(\tau_{ij})}{\partial x_k}\right]$$ with $$D = 0.5C_\mu F_1 + \frac{2}{3}0.22(1 - F_1)$$

Turbulence Models, cont'd
SSG/LRR RSM

Blending equation for $\phi = \alpha_\omega, \beta_\omega, \sigma_\omega, \sigma_d$:

$$\phi = F_1 \phi^{(\omega)} + (1 - F_1)\phi^{(\varepsilon)} \qquad\qquad F_1 = \tanh(\zeta^4)$$

$$\zeta = min\left[max\left(\frac{\sqrt{\tilde{k}}}{C_\mu \omega d}, \frac{500\mu}{\bar{\rho}\omega d^2}\right), \frac{4\sigma_\omega^{(\varepsilon)}\bar{\rho}\tilde{k}}{\sigma_d^{(\varepsilon)}\frac{\bar{\rho}}{\omega}max\left(\frac{\partial\tilde{k}}{\partial x_k}\frac{\partial\omega}{\partial x_k}, 0\right)d^2}\right]$$

Blending and closure coefficients for SSG/LRR RSM

	α_ω	β_ω	σ_ω	σ_d	C_1	C_1^*	C_2	C_3	C_3^*	C_4	C_5	D
LRR (ω)	0.5556	0.075	0.5	0	1.8	0	0	0.8	0	$\frac{(9C_2^{LRR}+6)}{11}$	$\frac{(-7C_2^{LRR}\,10}{11}$	$0.75C_\mu$
SSG (ε)	0.44	0.0828	0.856	1.712	1.7	0.9	1.05	0.8	0.65	0.625	0.2	0.22

Turbulence Models, cont'd
Wilcox RSM

$$\frac{\partial(\bar{\rho}\tau_{ij})}{\partial t}+\frac{\partial\left(\bar{\rho}\tau_{ij}\,\tilde{u}_k\right)}{\partial x_k}=-\bar{\rho}P_{ij}-\bar{\rho}\Pi_{ij}+\frac{2}{3}\beta^*\bar{\rho}\omega k\delta_{i,j}+\frac{\partial}{\partial x_k}\left[(\bar{\mu}+\sigma^*)\frac{\partial\tau_{ij}}{\partial x_k}\right]$$

$$\frac{\partial\bar{\rho}\omega}{\partial t}+\frac{\partial(\overline{\rho}\omega\tilde{u}_j)}{\partial x_j}=\alpha\frac{\bar{\rho}\omega}{k}\tau_{ij}\frac{\partial\tilde{u}_i}{\partial x_j}-\beta\bar{\rho}\omega^2+\sigma_d\frac{\bar{\rho}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j}+\frac{\partial}{\partial x_k}\left[(\bar{\mu}+\sigma\mu_T)\frac{\partial\omega}{\partial x_k}\right]$$

$$\Pi_{ij}=\beta^*\hat{C}_1\omega\left(\tau_{ij}+\frac{2}{3}k\delta_{ij}\right)-\hat{\alpha}\left(P_{ij}-\frac{2}{3}P\delta_{ij}\right)-\hat{\beta}\left(D_{ij}-\frac{2}{3}P\delta_{ij}\right)-\hat{\gamma}k\left(S_{ij}-\frac{1}{3}S_{kk}\delta_{ij}\right)$$

with,

$$P=\frac{1}{2}P_{kk}\qquad \mu_T=\bar{\rho}k/\omega\qquad D_{ij}=\tau_{ik}\frac{\partial\tilde{u}_k}{\partial x_j}+\tau_{jk}\frac{\partial\tilde{u}_k}{\partial x_i}$$

Closure coefficients for Wilcox RSM

$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{C}_1$	$\hat{C}_2$	α	β	β^*	σ	σ^*	β_o
$(8+C_2)/11$	$(8-C_2)/11$	$(60C_2-4)/55$	$\frac{9}{5}$	$\frac{10}{19}$	$\frac{13}{25}$	$\beta_o f_\beta$	$\frac{9}{100}$	0.5	0.6	0.0708

Turbulence Models, cont'd
Wilcox RSM, cont'd

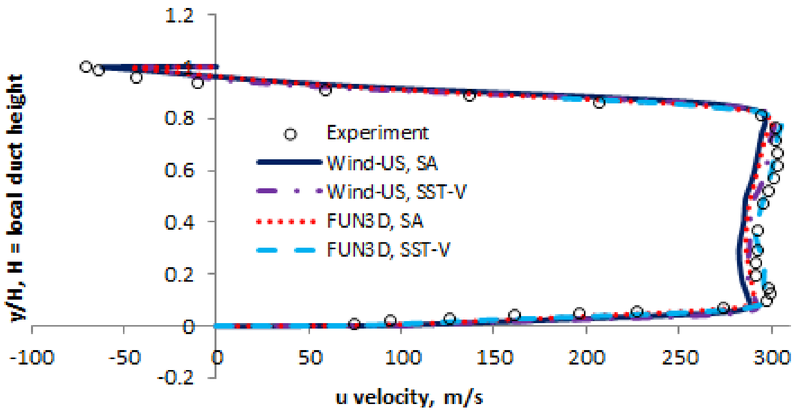
$$\sigma_d = \begin{cases} 0, & \dfrac{\partial k}{\partial x_j}\dfrac{\partial \omega}{\partial x_j} \le 0 \\ \dfrac{1}{8}, & \dfrac{\partial k}{\partial x_j}\dfrac{\partial \omega}{\partial x_j} > 0 \end{cases}$$

$$f_\beta = \frac{1 + 85\mathrm{X}_\omega}{1 + 100\mathrm{X}_\omega} \qquad \mathrm{X}_\omega = \left|\frac{W_{ij}\, W_{jk}\, \hat{S}_{ki}}{(\beta^* \omega)^3}\right| \qquad \hat{S}_{ki} = S_{ki} - \frac{1}{2}\frac{\partial \bar{u}_m}{\partial x_m}\delta_{k,i}$$

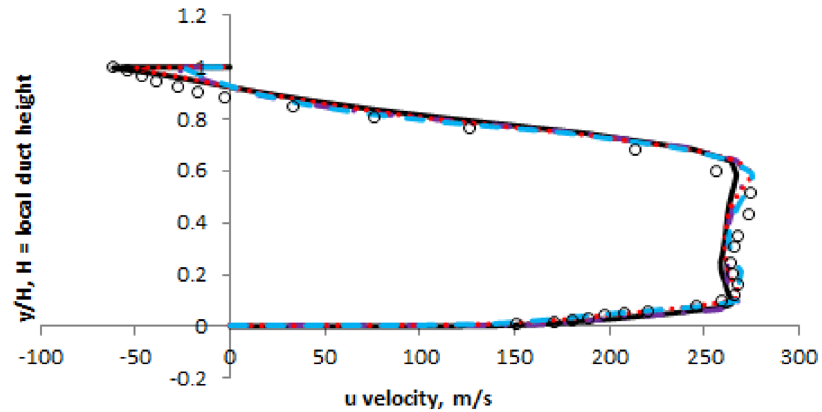
Test Cases
Sajben Diffuser – Strong Shock Case

Wind-US and FUN3D Results
Velocity

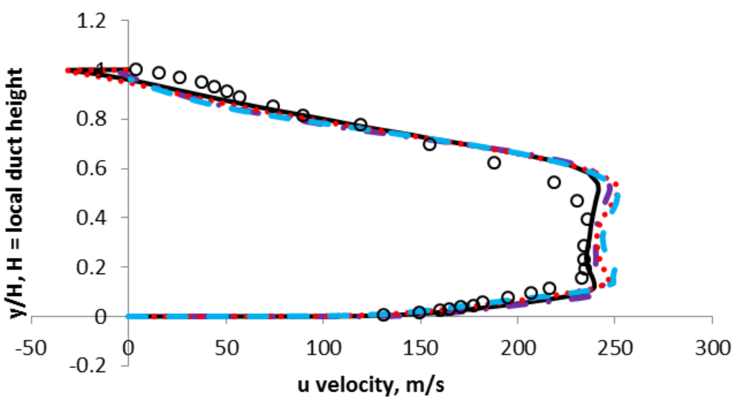
x/H = 2.9



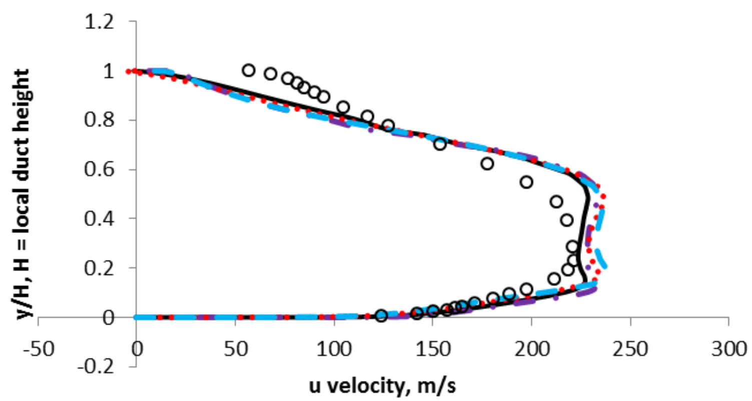
x/H = 4.6



x/H=6.4



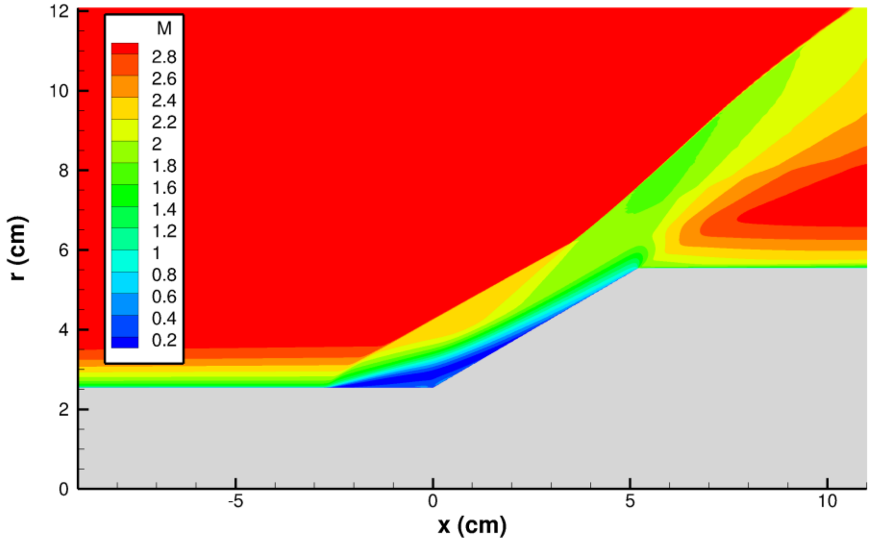
x/H=7.5



Test Cases

30º Axisymmetric Compression Corner

Mach Contours



Pressure Contours – Close-up of Corner